CUET Mathematics Solved Paper-2023

Held on 23 May 2023, (Shift-III)

SECTION: COMMON

1. In the context of differential equation Match List-I with List-II

List-I		List-II
A.	$\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$	I. Not a differential
B.	$x^2\frac{dy}{dx}=x^2-2y^2+xy$	II. Linear first order
C.	$\sin x+y=\cos(x+y)$	III. Variable separable
D.	$(x+y)\frac{dy}{dx}=1$	IV. Homogenous

Choose the correct answer from the options given below:

(a) A-I, B-II, C-III, D-IV (b) A-II, B-IV, C-III, D-I

(c) A-III, B-IV, C-I, D-II (d) A-IV, B-I, C-III, D-II

2. If $5x+y\le 100, x+y\le 60, x\ge 0, y\ge 0$. Then one of the corner points of the feasible region is :

(a) (60, 0) (b) (0, 100)

(c) (10, 50) (d) (0, 20)

3. Let A be a square matrix of order 3 then $|3A|$ is equal to

(a) $3|A|$ (b) $3^2|A|$

(c) $|A|^3$ (d) $3^3|A|$

4. Which of the following differential equation represents the family of circles touching the x-axis at the origin?

(a) $(x^2-y^2)dy-2xy\,dx=0$

(b) $(x^2+y^2)dy+2xy\,dx=0$

(c) $(x^2-y^2)dx+2xy\,dy=0$

(d) $(x^2+y^2)dy-2xy\,dx=0$

5. The critical points of $f(x)=x^3+x^2+x+1$ are

(a) 2, 1 (b) −2, −1

(c) 2, −1 (d) do not exist

6. Let $A=\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}$, then adjoint (A) is:

(a) $\begin{bmatrix}3 & 2\\ -1 & 4\end{bmatrix}$ (b) $\begin{bmatrix}4 & -1\\ 2 & 3\end{bmatrix}$

(c) $\begin{bmatrix}-4 & 1\\ -2 & -3\end{bmatrix}$ (d) $\begin{bmatrix}4 & 1\\ -2 & 3\end{bmatrix}$

7. The inverse of the matrix $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ is:

(a) $\begin{bmatrix}-2 & 3\\ 1 & -2\end{bmatrix}$ (b) $\begin{bmatrix}2 & -3\\ -1 & 2\end{bmatrix}$

(c) $\begin{bmatrix}-2 & -3\\ -1 & 2\end{bmatrix}$ (d) $\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

8. The integral $\int e^x\left(\frac{x-1}{2x^2}\right)dx$ is equal to:

(a) $\frac{e^x}{x}+C$, where C is constant of integration

(b) $\frac{e^x}{2x}+C$, where C is constant of integration

(c) $e^x x+C$, where C is constant of integration

(d) x^2e^x+C, where C is constant of integration

9. If $y=x^x$, $\frac{dy}{dx}$ will be:

(a) x^x (b) $x^x(1+\log x)$

(c) x^{x-1} (d) x^{x+1}

10. Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Then variance of the number of kings is

(a) $\frac{680}{(221)^3}$ (b) $\frac{6080}{(221)^2}$

(c) $\frac{680}{221}$ (d) $\frac{6800}{(221)^2}$

11. If a fair coin is tossed 10 times, then the probability of obtaining at least one head is:

(a) $\frac{1}{1024}$ (b) $\frac{17}{1024}$

(c) $\frac{1023}{1024}$ (d) $\frac{23}{1024}$

12. In a LPP, let R be the feasible region.

A. If R is unbounded then a max. /min. value of objective function may not exist.

B. If R is bounded then a max. and min. value of objective function will always exist.

C. If a solution exists, it must occur at a corner point.

D. If R is bounded then max. will exist but min. may or may not exist for an objective function.

Choose the correct answer from the options given below:

(a) A, B, C only (b) B only

(c) A, C only (d) D, C only

13. The order of a null matrix is:

(a) 0 (b) 1

(c) 2 (d) any order

14. The value of $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$ is:

(a) $\frac{x^2}{2}+\log|x|+C$, (where C is constant of integration)

(b) $\frac{x^2}{2}+\log|x|+2x+C$, (where C is constant of integration)

(c) $\frac{3}{2}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)+C$, (where C is constant of integration)

(d) $\frac{2}{3}x^{\frac{3}{2}}+2\sqrt{x}+C$, (where C is constant of integration)

15. The volume of a cube is increasing at the rate of $27 cm^3/s$. How fast is the surface area increasing when the length of the cube is 12 cm?

(a) $9 \ cm^2/s$ (b) $\frac{9}{4} cm^2/s$

(c) $\frac{4}{9} cm^2/s$ (d) $\frac{9}{2} cm^2/s$

SECTION: CORE MATHEMATICS

1. If $\begin{vmatrix} 2x & 2 \\ 4 & x \end{vmatrix} = 10$, then x is:

(a) ± 2 (b) ± 3

(c) ± 4 (d) 0

2. Which one of the following options is incorrect? For a square matrix A in the matrix equation AX= B.

(a) If $|A| \neq 0$, then there exists a unique solution

(b) If $|A| = 0$ and $(\text{adj } A)B \neq 0$ then there is no solution

(c) If $|A| \neq 0$ and $(\text{adj } A)B \neq 0$ then there is no solution

(d) If $|A| = 0$ and $(\text{adj } A)B = 0$ then system has infinitely many solutions

3. The area enclosed between the curves $y = x^2$ and $x = y^2$ is

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

4. The simplest form of $\tan^{-1}\left\{\frac{x}{\sqrt{a^2-x^2}}\right\}$ is, where $-a<x<a$.

(a) $\tan^{-1}\frac{x}{a}$ (b) $\tan^{-1}(ax)$

(c) $a\tan^{-1}\frac{x}{a}$ (d) $\sin^{-1}\frac{x}{a}$

5. If $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx}=$

(a) $-\frac{2}{1+x^2}$ (b) $\frac{2}{1+x^2}$

(c) $-\frac{1}{1+x^2}$ (d) $\frac{1}{1+x^2}$

6. The minimum value of $f(x) = |2x-1|$ is

(a) $-\infty$ (b) 0

(c) $\frac{1}{2}$ (d) 1

7. The solution of $y' - y'' = 2x$ is:

A. $y = x^2 + 2x + 2$ B. $y = x^2 + 2x + 1$

C. $y = x + 2$ D. $y = x^2 - 2x + 1$

Choose the correct answer from the options given below:

(a) A and B only (b) B only

(c) C only (d) A and D only

8. The value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular is:

(a) $-\frac{70}{11}$ (b) $\frac{70}{11}$

(c) $\frac{11}{70}$ (d) $-\frac{11}{70}$

9. If a fair coin is tossed 10 times the probability of atleast 6 heads is:

(a) $\frac{105}{512}$ (b) $\frac{53}{128}$

(c) $\frac{53}{64}$ (d) $\frac{193}{512}$

10. The unit vector in the direction of $\vec{a}+\vec{b}$ if $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ & $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$ is:

(a) $\hat{i}+0\hat{j}+\hat{k}$ (b) $\hat{i}-\hat{j}+\hat{k}$

(c) $\hat{i}+\hat{j}+\hat{k}$ (d) $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$

11. A doctor is to visit a patient. It is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10},\frac{1}{5},\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4},\frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he arrives late. The probability that he comes by bus is:

(a) $\frac{4}{9}$ (b) $\frac{1}{18}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

12. If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ & $\vec{b}=\hat{i}-3\hat{j}+5\hat{k}$ the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is:

(a) 60° (b) 90°

(c) 120° (d) 30°

13. Let A be the square matrix of order 3, then $|kA|$, where k is a scalar, is equal to:

(a) $3k|A|$ (b) $k^3|A|$

(c) $k^2|A|$ (d) $k|A|$

14. If x, y & z are non-zero real numbers, the inverse of matrix

$$A=\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \text{ is:}$$

(a) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(b) $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(c) $\frac{1}{xyz}\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(d) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

15. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{dx}{1+\sqrt{\tan x}}$ is equal to:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

16. The slope of the normal to the curve $y=2x^2-4$ at P(1, –2) is:

(a) 4 (b) –4

(c) $-\frac{1}{4}$ (d) 0

17. The maximum value of $\sin x + \cos x, x \in R$ is:

(a) 2 (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) Not known

18. The intervals for which $f(x) = x^4 - 2x^2$ is increasing are:

(a) $(-\infty, 1)$ (b) $(-1, \infty)$

(c) $(-\infty, -1) \cup (0, 1)$ (d) $(-1, 0) \cup (1, \infty)$

19. The relation $R = \{(a, b): a \le b^2\}$ on the set of real numbers is:

(a) Reflexive and symmetric

(b) Neither reflexive nor symmetric

(c) Transitive

(d) Reflexive but not symmetric

20. Points of discontinuity of the greatest integer function $f(x) = [x]$, where $[x]$ denotes integer less than or equal to x, are

(a) all natural numbers (b) all rational numbers

(c) all integers (d) all real numbers

21. The number of all onto functions from the set $\{1, 2, ...n\}$ to itself is

(a) 2^n (b) n^2

(c) $n!$ (d) $(2n)!$

22. The value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(a) 0 (b) $(a+b+c)$

(c) $(a-b)(b-c)(c-a)$ (d) $a^2+b^2+c^2$

23. The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{6}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2\pi}{3}$

24. The simplest form of $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x \ne 0$ is:

(a) $\tan^{-1}x$ (b) x

(c) $\frac{1}{2}\tan^{-1}x$ (d) $\sqrt{1+x^2}$

25. The general solution of $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ is:

(given that C is the constant of integration)

(a) $\tan^{-1}x = y + \frac{y^3}{3} + C$

(b) $\tan^{-1}y = x + \frac{x^3}{3} + C$

(c) $\tan^{-1}x = \tan^{-1}y + C$

(d) $\tan^{-1}x = \tan^{-1}y = C$

26. If $y = x^{(x \sin x)}$ then $\frac{dy}{dx} = ?$

(a) $x^{x \cos x}$

(b) $x^{(x \sin x)}[\sin x + \sin \log x]$

(c) $x^{(x \sin x)}[\sin(1 + \log x) + x \log x \cos x]$

(d) $x^{(x \sin x)}[\sin \log x + x \log x \cos x]$

27. The area of the region $\{(x, y): y \ge x^2 \text{ and } y \le |x|\}$ is

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

28. The angle between the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{j} - 2\hat{k} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ is:

(a) $\sin^{-1}\left(\frac{19}{21}\right)$ (b) $\cos^{-1}\left(\frac{19}{23}\right)$

(c) $\cos^{-1}\left(\frac{19}{21}\right)$ (d) $\sin^{-1}\left(\frac{19}{23}\right)$

29. The matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a

(a) zero matrix (b) identity matrix

(c) scalar matrix (d) diagonal matrix

30. Match List I with List II

List-I	List-II
A. $lx + my + nz = d$ is	I. Equation of plane passing through a given point and normal to given vector
B. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is	II. Equation of plane
C. $(\vec{r} - \vec{a}).\vec{n} = 0$	III. Plane passing through the intersection of two planes
D. $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$	IV. Intercept form of

Choose the correct answer from the options given below:

(a) A-I, B-III, C-IV, D-II (b) A-IV, B-III, C-I, D-II

(c) A-II, B-IV, C-I, D-III (d) A-I, B-II, C-III, D-IV

31. The value of integral $\int \sqrt{4x^2+9}\, dx$ is

(a) $\frac{x}{2}\sqrt{4x^2+9}+\frac{9}{2}\log\left|2x+\sqrt{4x^2+9}\right|+C$

(b) $\frac{x}{2}\sqrt{4x^2+9}+\frac{3}{2}\log\left|2x+\sqrt{4x^2+9}\right|+C$

(c) $2x\sqrt{4x^2+9}+\frac{9}{2}\log\left|2x+\sqrt{4x^2+9}\right|+C$

(d) $x\sqrt{4x^2+9}+\frac{9}{4}\log\left|2x+\sqrt{4x^2+9}\right|+C$

32. Let $f(x)=x^3$ be a function with domain {0, 1, 2, 3} then domain of f^{-1} is:

(a) {3, 2, 1, 0} (b) {0, –1, –2, –3}

(c) {0, 1, 8, 27} (d) {0, –1, –8, –27}

33. The area of the parallelogram determined by the vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$ is

(a) $8\sqrt{3}$ (b) $4\sqrt{3}$

(c) $16\sqrt{3}$ (d) $2\sqrt{3}$

34. A manufacturing company makes two models M_1 and M_2 of a product. Each piece of M_1 requires 9 labour hours for fabricating and one labour hour for finishing. Each piece of M_2 require 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of ₹ 800 on each piece of M_1 and ₹ 1200 on each peice of M_2

The above Linear Programming Problem [LPP] is given by

(a) Maximize $Z=800x+1200y$
Subject to constraints,
$3x+4y \le 60$
$x+3y \le 30$
$x, y \ge 0$

(b) Maximize $Z=800x+1200y$
Subject to constraints,
$3x+4y \ge 60$
$x+3y \ge 30$
$x, y \ge 0$

(c) Minimize $Z=800x+1200y$
Subject to constraints,
$3x+4y \le 60$
$x+3y \ge 30$
$x, y \ge 0$

(d) Minimize $Z=800x+1200y$
Subject to constraints,
$3x+4y \ge 60$
$x+3y \le 30$
$x, y \ge 0$

35. A manufacturing company makes two models M_1 and M_2 of a product. Each piece of M_1 requires 9 labour hours for fabricating and one labour hour for finishing. Each piece of M_2 require 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of ₹ 800 on each piece of M_1 and ₹ 1200 on each piece of M_2

The maximum profit will be at the point

(a) (0,10) (b) (20,0)

(c) (12,6) (d) (0, 0)

Hints & Explanations

1. **(d)** (A) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \Rightarrow \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

Which is a variable separable.

(B) $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy \Rightarrow \frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$

Which is a homogeneous differential equation.

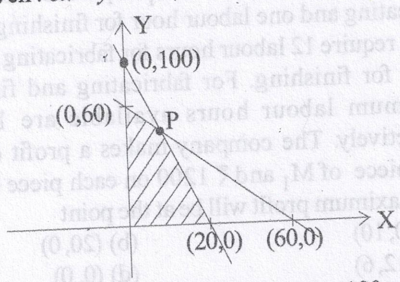
(C) sin $(x + y)$ = cos $(x + y)$ → not a differential equation.

(D) $(x+y)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dx}{dy} = x + y \Rightarrow \frac{dx}{dy} - x = y$

Which is a linear differential equation.

2. **(c)** Given : $5x + y \leq 100,\ x + y \leq 60,\ x \geq 0,\ y \geq 0.$



For point P, solve equation $5x + y = 100$ and $x + y = 60$

$\Rightarrow 5x + 60 - x = 100 \Rightarrow 4x = 40 \Rightarrow x = 10$

$\therefore y = 60 - 10 = 50.$

$\therefore$ Co-ordinate of P is (10, 50), which is a corner point

3. **(d)** $\because$ A is square matix of order 3

$\therefore |3A| = 3^3|A|$

4. **(a)** Euation of circles touching the x-axis at original is

$(x - 0)^2 + (y - k)^2 = k^2$

$\Rightarrow x^2 + y^2 - 2yk = 0 \Rightarrow k = \frac{1}{2y}\left[x^2 + y^2\right]$

Differentiating with respect to 'x' . we get

$2x + 2y\frac{dy}{dx} - 2k\frac{dy}{dx} = 0.$

$\Rightarrow x + y\frac{dy}{dx} - k\frac{dy}{dx} = 0.$

$\Rightarrow x + \frac{dy}{dx}\left[y - \frac{1}{2y}\left(x^2 + y^2\right)\right] = 0$

$\Rightarrow 2xy + \frac{dy}{dx}\left[y^2 - x^2\right] = 0$

$\Rightarrow \left(x^2 - y^2\right)\frac{dy}{dx} - 2xy = 0$

5. **(d)** $f(x) = x^3 + x^2 + x + 1$

$\Rightarrow f'(x) = 3x^2 + 2x + 1$

For critical points: $f'(x) = 0$

$\Rightarrow 3x^2 + 2x + 1 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4-12}}{2}$

$\Rightarrow x = \frac{-2 \pm \sqrt{-8}}{2}$ (not possible)

So, critical point does not exist.

6. **(d)** Here, $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$

odj $(A) = \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}$

7. **(b)** The given matrix is

$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Then $|A| = 2 \times 2 - 3 \times 1 = 1$

and adj$(A) = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{1}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

8. **(d)** Let $I = \int e^x \left(\frac{x-1}{2x^2}\right) dx.$

$= \frac{1}{2}\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx.$

$= \frac{1}{2}\int \left[e^x \cdot \frac{1}{x}\right] + C.$

$= \frac{e^x}{2x} + C.$

9. **(b)** Given, $y = x^2$

Taking log on both sides:

$\log y = x \log x$

Differentiating both sides, we get:

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x.$

$\Rightarrow \frac{dy}{dx} = y(1 + \log x)$

$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$

10. (d) Probability of getting O kings,

$$P(X=0)=\frac{{}^4C_0\times{}^{48}C_2}{{}^{52}C_2}=\frac{1128}{1326}$$

Probability of getting 2 kings $P(X=1)$

$$=\frac{{}^4C_1\times{}^{48}C_1}{{}^{52}C_2}=\frac{192}{1326}$$

Probability of getting 2 kings $P(X=2)$

$$=\frac{{}^4C_2\times{}^{48}C_0}{1326}=\frac{6}{1326}$$

Probability distribution is

X	0	1	2
$P(X)$	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

$$E(X)=0\times\frac{1128}{1326}+1\times\frac{192}{1326}2\times\frac{6}{1326}=\frac{34}{221}$$

$$E(X^2)=0^2\times\frac{1128}{1326}+1^2\times\frac{192}{1326}4\times\frac{6}{1326}=\frac{36}{221}$$

$$\text{Variance}=E(X^2)-[E(X)]^2=\frac{36}{221}-\left(\frac{34}{221}\right)^2$$

$$=\frac{6800}{(221)^2}$$

11. (c) Probability of not getting head $=\frac{1}{2}$

When coin is tossed 10 times then probability of not getting head $P=\left(\frac{1}{2}\right)^{10}$

Probability of obtaining at least one head $=1-P$

$$=1-\left(\frac{1}{2}\right)^{10}=\frac{1023}{1024}$$

12. (a) If R be feasible region in an LPP then-

(A) If R is unbounded then a max/min value of objective function may not exist.

(B) If R is bounded then a mix/min value of objective function will always exist.

(C) If a solution exists, it must occur at a corner point.

13. (d) Null matrix can have any order.

14. (b) Let $I=\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

$$=\int\left(x+\frac{1}{x}+2\right)dx.$$

$$=\frac{x^2}{2}+\ell n|x|+2x+C.$$

15. (a) Let x be the side of the cube.

Given, $\frac{dv}{dt}=27\text{ cm}^3/\text{s}$

$$\Rightarrow\frac{d(x^3)}{dt^2}=27\text{ cm}^3/\text{s}$$

$$\Rightarrow 3x^2\frac{dx}{dt}=27\text{ cm}^3/\text{s}$$

$$\Rightarrow\frac{dx}{dt}=\frac{9}{x^2}$$

Surface area of the cube $S=6x^2$

$$\Rightarrow\frac{dS}{dt}=12x\frac{dx}{dt}=12x\times\frac{9}{x^2}$$

$$=\frac{108}{x}\qquad\{\because x=120\}$$

$$=\frac{108}{12}=9\text{ cm}^2/\text{sec}$$

Section : Core Mathematics

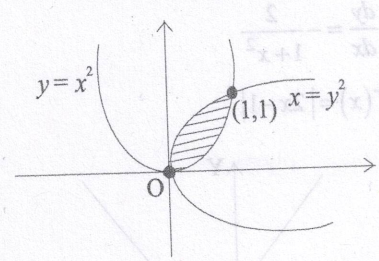
1. (b) $\because\begin{vmatrix}2x & 2\\ 4 & x\end{vmatrix}=10\Rightarrow 2x^2-8=10$

$$\Rightarrow 2x^2=18\Rightarrow x^2=9\Rightarrow x=\pm3$$

2. (c) If $|A|\neq0$ and adj $(A)B\neq0$

$\Rightarrow$ The matrix equation AX = B will have unique solution.

3. (c) Let A be the area between the curves.



Solving $x=y^2$ and $y=x^2$, we get point P (1, 1)

Then, $A=\int_{x=o}^{1}(y_1-y_2)\,dx.$

$$=\int_o^1\left(\sqrt{x}-x^2\right)dx=\left[\frac{2}{3}\times^{3/2}-\frac{x2}{3}\right]_0^1$$

$$=\frac{2}{3}\times1-\frac{1}{3}=\frac{1}{3}$$

4. (d) $I=\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

Let $\frac{x}{a} = \sin\theta \Rightarrow \cos\theta = \sqrt{1-\frac{x^2}{a^2}} = \frac{1}{a}\sqrt{a^2-x^2}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{x}{a}}{\frac{1}{a}\sqrt{a^2-x^2}} = \frac{x}{\sqrt{a^2-x^2}}$$

$$\Rightarrow I = \tan^{-1}(\tan\theta) = \theta$$

$$\Rightarrow I = \sin^{-1}\left(\frac{x}{a}\right) \qquad \left\{\because \frac{x}{a} = \sin\theta\right\}$$

5. **(a)** $\because y = \sin^{-1}\left(\frac{1-x^2}{1-1x^2}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Let, $x = \tan\theta$

$$\frac{1-x^2}{1+x^2} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$\frac{1-x^2}{1+x^2} = \frac{\cos 2\theta}{1} = \cos 2\theta$$

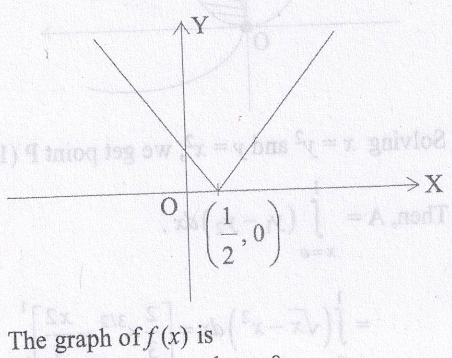
$$\Rightarrow \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}(\cos 2\theta) = 2\theta$$

$$\Rightarrow \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x \qquad \{\because x = \tan\theta\}$$

$$\therefore y = \frac{\pi}{2} - 2\tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2}{1+x^2}$$

6. **(b)** $\therefore f(x) = |2x-1|$



The graph of $f(x)$ is

So, the minimum value = 0.

7. **(a)** (A) $y = x^2 + 2x + 2$

$y' = 2x + 2 \quad y'' = 2$

$y' - y'' = 2x + 2 - 2 = 2x$

This is a the solution.

(B) $y = x^2 + 2x + 1$

$y' = 2x + 2 \quad y'' = 2$

$y' - y'' = 2x + 2 - 2 = 2x$

This is also a solution.

(C) $y = x + 2 \Rightarrow y' = 1$ and $y'' = 0$

$y' - y'' \Rightarrow 1 - 0 = 1 \neq 2x$

This is not the solution.

(D) $y = x^2 - 2x + 1$

$y' = 2x - 2 \quad y'' = 2$

$y' - y'' = 2x - 2 - 2 = 2x - 4 \neq 2x$

This is also not the solution.

8. **(b)** Equation of line in standard form

$$\frac{x-1}{-3} = \frac{y-2}{\frac{2\lambda}{7}} = \frac{z-3}{2} \quad \text{...(1)}$$

$$\text{and } \frac{x-1}{\frac{-3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} \quad \text{...(2)}$$

Since, line (1) and (2) are perpendicular.

$$(-3)\left(-\frac{3\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right) \times 1 + 2 \times (-5) = 0$$

$$\frac{9\lambda + 2\lambda}{7} = 10 \Rightarrow \lambda = \frac{70}{11}$$

9. **(d)** Probability of at least 6 heads

$$P(x \geq 6) = p(x=6) + p(x=7) + p(x=8) + p(x=9) + p(x=10)$$

$$= {}^{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 + {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10}\left[{}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$$

$$= \frac{386}{1024} = \frac{193}{512}$$

10. **(d)** $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \quad \vec{b} = -\hat{i} + \hat{j} - \hat{k}$

$\vec{a} + \vec{b} = \hat{i} + 0\hat{j} + \hat{k}$

$|\vec{a} + \vec{b}| = \sqrt{1+0+1} = \sqrt{2}$

The unit in the direction of $\vec{a} + \vec{b}$ is

$$\hat{n} = \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{\hat{i} + \hat{k}}{\sqrt{2}}$$

$$\hat{n} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

11. **(a)** Let E_1, E_2, E_3 and E_4 are the events that doctor come by train, bus, scoter by their means of transport.

Let E be the event that doctor arrives late

Then, we have

$P(E_1)=\frac{3}{10},\quad P(E_2)=\frac{1}{5}$

$P(E_3)=\frac{1}{10},\quad P(E_4)=\frac{2}{5}$

$P(E|E_1)=\frac{1}{4},\quad P(E|E_2)=\frac{1}{3},$

$P(E|E_3)=\frac{1}{12},\quad P(E|E_4)=0$

So, the required probability is $P\left(\frac{E_2}{E}\right)$

$$=\frac{P(E_2)\times P(E|E_2)}{P(E_1)P(E|E_1)+P(E_2)P(E|E_2)P(E_3)P(E|E_3)+P(E_4)P(E|E_4)}$$

$$\Rightarrow P\left(\frac{E_2}{E}\right)=\frac{\frac{1}{5}\times\frac{1}{3}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times 0}=\frac{4}{9}$$

12. **(b)** Given: $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}\quad \vec{b}=\hat{i}-3\hat{j}+5\hat{k}$

$\vec{a}+\vec{b}=6\hat{i}-4\hat{j}+2\hat{k}\quad \vec{a}-\vec{b}=4\hat{i}+2\hat{j}-8\hat{k}$

$|\vec{a}+\vec{b}|=\sqrt{36+16+4}=\sqrt{56}$

$|\vec{a}+\vec{b}|=\sqrt{4+16+64}=\sqrt{84}$

Then, $\cos\theta=\frac{\left|(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})\right|}{|\vec{a}+\vec{b}|\cdot|\vec{a}-\vec{b}|}=\frac{|24-8-16|}{\sqrt{56}\sqrt{84}}$

$\Rightarrow \cos\theta=0\Rightarrow \theta=\frac{\pi}{2}=90°$

13. **(b)** Since A is the square matrix of order 3

Then $|kA|=k^3|A|$

14. **(a)** Given, $A=\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}$

$\because \begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}\begin{bmatrix}x^{-1}&0&0\\0&y^{-1}&0\\0&0&z^{-1}\end{bmatrix}$

$=\begin{bmatrix}x^{-1}&0&0\\0&y^{-1}&0\\0&0&z^{-1}\end{bmatrix}\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$\therefore A^{-1}=\begin{bmatrix}x^{-1}&0&0\\0&y^{-1}&0\\0&0&z^{-1}\end{bmatrix}$

15. **(b)** Let $I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ ...(1)

$$I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\cot x}}$$

$$\Rightarrow I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\tan x}}{\sqrt{\tan x}+1}dx \quad ...(2)$$

Adding (1) and (2) we get

$$\Rightarrow 2I=\int_{\pi/6}^{\pi/3}(1)\,dx=[x]_{\pi/6}^{\pi/3}=\left[\frac{\pi}{3}-\frac{\pi}{6}\right]=\frac{\pi}{6}$$

$\Rightarrow I=\frac{\pi}{12}$

16. **(c)** The given curve is

$\because y=2x^2-4\Rightarrow \frac{dy}{dx}=4x-0=4x$

$\left(\frac{dy}{dx}\right)_{(1,-2)}=4$

Slope of normal $=-\frac{1}{\left(\frac{dy}{dx}\right)_{(1,-2)}}=-\frac{1}{4}$

17. **(b)** The maximum value of $a\sin x+b\cos x$ is $\sqrt{a^2+b^2}$

Then maximum value of $\sin x+\cos x=\sqrt{1^2+1^2}=\sqrt{2}$

18. **(d)** Since, $f(x)=x^4-2x^2$

$f(x)$ is increasing if

$f'(x)>0\Rightarrow 4x^3-4x>0$

$\Rightarrow x(x^2-1)>0$

$\Rightarrow x\in(-1,0)\cup(1,\infty)$

19. **(d)** Since, $R=\{(a,b):a\le b^2\}$

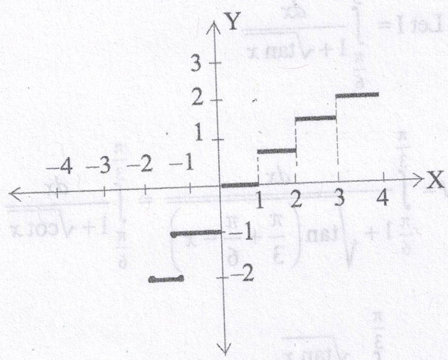
$\because a\le a^2\Rightarrow (a,a)\in R$

So, R is reflexive.

$\because 1\le 3^2$ But $3\not\le 1^2$

So, R is not symmetric.

20. **(c)** The graph of $f(x) = [x]$ is



So $f(x)$ is discontinuous on all the integral point.

21. **(c)** Let $X = \{1, 2, \ldots n\}$

No of elements in $X = n$

No of into functions from X to $X = n\,!$

22. **(a)** $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

$R_2 \to R_1 - R_2,\quad R_3 \to R_2 - R_3$

$$= \begin{vmatrix} 1 & a & b+c \\ 0 & a-b & b-a \\ 0 & b-c & c-b \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 1 & a & b+c \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= (a-b)(b-c)[1(-1+1)+0+0] = 0$$

23. **(d)** $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \cot^{-1}(\cot(90+30))$

$$= 120° = \frac{2\pi}{3}$$

24. **(c)** $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Let $x = \tan\theta$

Then $\frac{\sqrt{1+x^2}-1}{x} = \frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta} = \frac{\sec\theta-1}{\tan\theta}$

$$= \frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\Rightarrow \frac{\sqrt{1+x^2}-1}{x} = \tan\frac{\theta}{2}$$

$$\Rightarrow \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$= \frac{1}{2}\tan^{-1}x \quad \{\because \tan\theta = x\}$$

25. **(b)** $\because \frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2 = (1+x^2)(1+y^2)$

$$\Rightarrow \frac{dy}{(1+y^2)} = (1+x^2)\,dx$$

Integrating both side, we get:

$$\tan^{-1} y = x + \frac{x^3}{3} + C.$$

26. **(c)** $\because y = x^{(x \sin x)}$

Taking log on both sides:

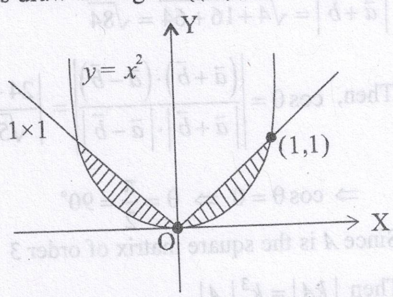
$\log y = x \sin x \cdot \log x$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = (x\sin x)\frac{1}{x} + (x\cos x + \sin x)\log x$$

$$\Rightarrow \frac{dy}{dx} = y[\sin x + (x\cos x + \sin x)\log x]$$

$$\Rightarrow \frac{dy}{dx} = x^{(x\sin x)}[\sin x(1+\log x) + x\log x\cos x]$$

27. **(d)** Let us draw the region $\{(x, y) : y \geq x^2 \text{ and } y \leq |x|\}$



The shaded region is the required area.

$$\therefore \text{Area} = 2\int_0^1 (y_1 - y_2)\,dx = 2\int_0^1 (x - x^2)\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{3}\right] = 2 \times \frac{1}{6}$$

$$= \frac{1}{3}$$

28. **(c)** The equation of lines are –

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ ...(1)

$\vec{r} = 5\hat{i} - 2\hat{k} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ ...(2)

Here, $\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

Angle between line (1) and (2) is

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \left|\frac{3+4+12}{\sqrt{1+4+4}\sqrt{9+4+36}}\right|$$

$$= \left|\frac{19}{3\times 7}\right| = \left|\frac{19}{21}\right|$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

29. **(d)** $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a diagonal matrix.

30. **(c)** (A) $lx + my\,nz = d \rightarrow$ Equation of plane in normal form.

(B) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is interapt form of the plane.

(C) $(\vec{r} - \vec{a})\hat{n} = 0$ is equation of plane passing through a given point and normal to given vector.

(D) $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$ is plane passing through the intersection of two planes.

31. **(Bonus)** Let $I = \int \sqrt{4x^2 + 9}\, dx$

$$= \int \sqrt{(2x)^2 + 3^2}\, dx$$

$$= \frac{1}{2\times 2}\left[2x\sqrt{(2x)^2 + 3^2} + 3^2 \log\left(2x + \sqrt{(2x)^2 + 3^2}\right)\right] + C$$

$$= \frac{1}{4}\left[2x\sqrt{4x^2+9} + 9\log\left(2x + \sqrt{4x^2+9}\right)\right] + C$$

$$= \frac{x}{2}\left[\sqrt{4x^2+9} + \frac{9}{4}\log\left(2x + \sqrt{4x^2+9}\right)\right] + C$$

32. **(c)** $\because f(x) = x^3$

Domain of $f(x)$ is $\{0, 1, 2, 3\}$

Range $f'(x) = \{0, 1, 8, 27\}$

Now, Domain of $f^{-1}(x) = \text{Range}\,(f(x))$
$= \{0, 1, 8, 27\}$

33. **(a)** Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

Area of the parallelogram = $|\vec{a} \times \vec{b}|$

$$\text{Now, } \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}(8) - \hat{j}(-8) + \hat{k}(-8)$$

$$= 8\hat{i} + 8\hat{j} - 8\hat{k}$$

Then, $|\vec{a}\times\vec{b}| = \sqrt{64+64+64} = 8\sqrt{3}$

$\therefore$ Area of parallelogram = $8\sqrt{3}$

34. **(a)** Suppose x is the number of pieces of Model M_1 and y is the number of pieces of Model M_2.

Then, total profit = $800x + 1200y$.

let $Z = 800x + 1200y$

We have the following mathematical model for the given problem $\Rightarrow$

Maximize $Z = 800x + 1200y$. ...(1)

Subject to the constraints:

from fabricating hours, $9x + 12y \le 180$

$\Rightarrow 3x + 4y \le 60$ ...(2)

from finishing hours, $x + 3y \le 30$ ...(3)

Also, non-negativity constraint. $x \ge 0$ $y \ge 0$ ...(4)

So, the *LPP* prob is

Maximize $Z = 800x + 1200y$

Subject to constraints,

$3x + 4y \le 60$

$x + 3y \le 30$

$x_1\, y \ge 0$

35. **(c)** Here, the *LPP* is,

Maximize, $Z = 800x + 1200y$ ...(1)

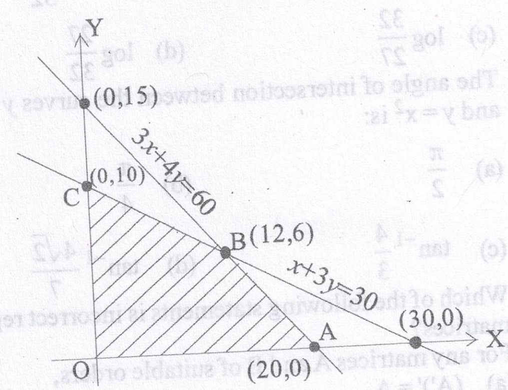
Subject to contraints

$3x + 4y \le 60$ ...(2)

$x + 3y \le 30$ ...(3)

$x_1\, y \ge 0$ ...(4)

The feasible region (OABC) can be found by the linear inequalities (2) to (4) as shown in the figure.



Here, the feasible region is bounded.

Now, let us evaluate the objective function Z at each corner point as shown below.

Corner point	Z=800x+1200y
O(0,0)	0
A(20,0)	16000
B(12,6)	16800 → maximum
C(0,10)	12000

$\therefore$ The maximum profit will be at the point (12, 6).